3.3: Higher-Order Linear, Homogeneous Equations with Constant Coefficients
Complex Roots

Theorem 3. (Complex Roots)
If the characteristic equation (2) has a complex root $r = a + ib$, then the part of a general solution of the differential equation (1) corresponding to r is of the form

$$e^{ax}(c_1 \cos bx + c_2 \sin bx).$$

Exercise 1. Find the particular solution to the initial value problem

$$y'' - 4y' + 5y = 0, \quad y(0) = 1, y'(0) = 5.$$

Exercise 2. Find the general solution to

$$y^{(4)} + 4y = 0.$$

Theorem 4. (Repeated Complex Roots)
If the characteristic equation (2) has a repeated complex root $r = a + ib$ of multiplicity k, then the part of a general solution of the differential equation (1) corresponding to r is of the form

$$\sum_{p=0}^{k-1} x^p e^{ax}(c_p \cos bx + d_p \sin bx).$$

Exercise 3. Find the general solution to

$$y^{(6)} - 12y^{(5)} + 63y^{(4)} - 184y^{(3)} + 315y'' - 300y' + 125y = 0$$

which has characteristic equation

$$(r^2 - 4r + 5)^3 = 0.$$